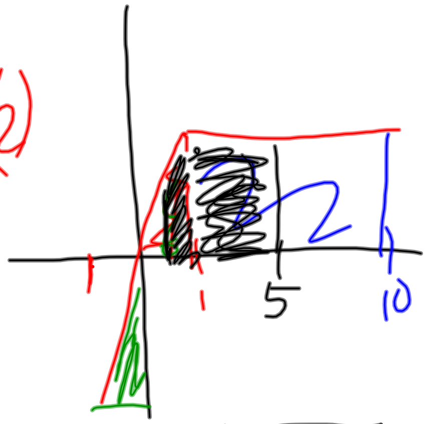
6.5/16 $f(x) = \begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases}$

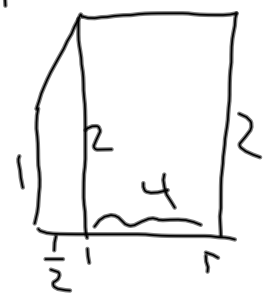
$$\lim_{\max \Delta x \to 0} \sum_{k=1}^{n} f(x_k^*)\Delta x_k$$



A) $\int_0^1 f(x)\,dx = A_\Delta = \frac{1}{2}bh = \frac{1}{2}(1)(2) = 1$

B) $\int_{-1}^{1} f(x)\,dx = 0$

C) $\int_1^{10} f(x)\,dx = 9 \cdot 2 = 18$



D) $\int_{1/2}^{5} f(x)\,dx = \frac{1}{2}(1+2)\left(\frac{1}{2}\right) + 2(4) = \frac{3}{4} + 8$

Limits

Derivatives

Definite Integral

$$f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*) \Delta x_k$$

Anti-Derivative

1 Ez
2 Ez
3 M
4 M
5 Ez
6 M
7 M
8 Ez.7
9 MH
10 H
11 H
12 H
13 H
14 H

Indeterminate Forms

A limit is an indeterminate form if

* substitution fails
* there are 2 parts that work against each other

$\frac{\infty}{\infty}$

$\frac{0}{0}$

$$\lim_{x \to 0} \frac{2x}{x}$$ inderminate of type $\frac{0}{0}$

$$\lim_{x \to \infty} \frac{x^2}{x+4}$$

suppose $x=100$

$$\frac{100^2}{104} \approx 96 \sim$$

$$\frac{120^2}{104} = 138 \approx$$

suppose

120

$$\frac{100^2}{124} = 80 \sim$$

* Strat. for Resolving $\frac{\infty}{\infty}$

* Strat. for Resolving $\frac{0}{0}$

* sim/diff between

$$\lim_{x \to ?} \frac{f(x)}{g(x)} \quad \text{and} \quad \lim_{x \to ?} \frac{f'(x)}{g'(x)}$$

L'Hospital's Rule

If, in $\lim\limits_{x \to ?} \frac{f(x)}{g(x)}$, I have a limit of indeterminate form type $\frac{0}{0}$ or type $\frac{\infty}{\infty}$

then $\lim\limits_{x \to ?} \frac{f(x)}{g(x)} = \lim\limits_{x \to ?} \frac{f'(x)}{g'(x)}$!

$$\lim_{x \to 0} \frac{2x}{x} = \lim_{x \to 0} \frac{2}{1} = 2$$

$$\lim_{x \to \infty} \frac{2x^2}{x} = \lim_{x \to \infty} \frac{4x}{1} = \infty$$

$$\lim_{x \to \infty} \frac{2x^3}{x^2} = \lim_{x \to \infty} \frac{6x^2}{2x}$$

$$= \lim_{x \to \infty} \frac{12x}{2} = \infty$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$$

$$\lim_{x\to\infty} \frac{\sin x}{\ln x} = 0$$

HW) 6, 12, 14-15, 17-18, 20, 26, 34

Limits

Derivatives

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

Anti-Derivatives

Definite Integrals

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(x_k^*)\,\Delta x_k$$

A limit is an indeterminate form

if

* substitution fails
* there are two pieces that work opposite to each other

$\frac{0}{0}$

$\frac{\infty}{\infty}$

$$\lim_{x\to\infty} \frac{x^2}{x+4}$$

assum

$x=100$

$$\frac{100^2}{104} \approx 96$$

assumethon

$x=120$

$$\frac{120^2}{104} \approx 138$$

$$\frac{100^2}{124} \approx 80.$$

Strategies for resolving $\frac{\infty}{\infty}$

* end behavior → ONLY to $\lim_{x\to\infty}$ or $\lim_{x\to-\infty}$

* cancel xs

* cancel infinities

strategies for resolv... $\frac{0}{0}$

* cancelling xs

* Cancel the zeros

compare

$\lim \frac{f(x)}{g(x)}$ $\qquad\qquad$ $\lim \frac{f'(x)}{g'(x)}$

L'Hospital's

L'Hospital's Rule

If a limit is an indeterminate form of type $\frac{\infty}{\infty}$ or type $\frac{0}{0}$

then

$$\lim_{x \to ?} \frac{f(x)}{g(x)} = \lim_{x \to ?} \frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 0} \frac{2x}{x} = \lim_{x\to 0} \frac{2}{1} = 2$$

$$\lim_{x\to \infty} \frac{2x}{x} = \lim_{x\to \infty} \frac{2}{1} = 2$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin(2x)}{\sin(5x)} = \lim_{x\to 0} \underbrace{\frac{\sin(2x)}{2x}}_{1} \cdot \underbrace{\frac{5x}{\sin(5x)}}_{1} \cdot \frac{2}{5} = \frac{2}{5}$$

$$= \lim_{x\to 0} \frac{2\cancel{\cos(2x)}^{\,1}}{5\cancel{\cos(5x)}_{\,1}} = \frac{2}{5}$$